Utilization of The Thrasher and Rice Mill Machines in Composition Function Learning: A Hypothetical Learning Trajectory Design

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Abstract

The study aims to design mathematics learning in composite function concepts with farm tools, which are thrasher and rice mill machines; this farm's tool is used as to starting point in the learning process. The research method used is design research with a preliminary design, design experiment, and analysis retrospective stages. This study describes the design of the thrasher and rice mill machine to facilitate a real contribution for student understanding of the composite function concept. The participant of this research is 10 eleventh-grade students from one of the senior high school in East Java. The results of this study reveal that students are able to make associations from the thrasher and rice mill machine through the determination of the input and output of the machines to the formula of the composite function concept. So, the stages in the learning trajectory have an important role in understanding the composition function concept from informal level to formal level and also make the study of mathematics more easy, simple, fun, and comfortable.

Keywords*: Thrasher and Rice Mill Machine, Composition Function, Hypothetical Learning Trajectory, Design Research*

INTRODUCTION

The utilization of local contexts that are easily accessible by students in their activities helps students understand mathematical concepts strongly. This situation happens because the local context helps to visualize the mathematical mental model that is sometimes too abstract for the student. According to Amsikan et al. and Ikhwanudin, Using the context of local wisdom in mathematics learning is expected to create a contextually based learning process that is close to students and strengthens the character of students (Deda & Maifa, 2021). Mathematics is a human activity, so mathematics is

connected with students' daily activities (Freudenthal, 2002). The development and implementation of mathematics in life is a form of the students learning process (Tanujaya et al., 2017; Wahyu et al., 2017).

Mathematics related to everyday life can be implemented formally and informally. In formal learning environments, teachers use several material and instructional approaches including: math word problems that illustrate realistic contexts (according to Widjaja), life-relevant references that directly exemplify mathematical concepts (according to Gainsburg) and hands-on activities to actively discover math concepts (according to Wijers, Jonker, and Drijvers); Children's mathematizing experiences also emerge during their play at home (according to Anderson; Wager and Parks) e.g., counting or sorting toys; in these informal settings, prior work suggests learners benefit from: directing attention to mathematics during real-life activities (according to Seo and Ginsburg); adult intervention to scaffold learning according to Lytle; and (iii) exploration through unstructured manipulation of objects (Kang et al., 2020).

Various studies have been carried out by integrating culture or daily activities into students' learning of mathematics, for example, the use of Kawung Batik pattern for learning transformation geometry (rotation) (Risdiyanti & Prahmana, 2018), batik pattern for learning reflection material (Novrika et al., 2016), card games in learning number operations (Prahmana, 2012), and rotating card game for learning Lowest Common Multiple (LCM) concept (Prahmana, 2010). All of the mathematics learning designs in the research that have been carried out are similar to the reform of mathematics education, namely the Indonesian Realistic Mathematics Education. Indonesian Realistic Mathematics Education (Read: PMRI) demands mathematics learning methods that are supporting active learning activities, focus on solving problems in everyday contexts, and find new formulations of mathematical concepts (K. Gravemeijer, 2010; Hadi, 2016; Soedjadi, 2007).

The presence of Indonesian Realistic Mathematics Education (PMRI) as one of the innovations in mathematics education in Indonesia corresponds to current conditions. PMRI is a manifestation of Realistic Mathematics Education which has been devoted to implementation in Indonesia. Characteristic of RME is that rich, "realistic" situations are given a prominent position in the learning process (Heuvel-Panhuizen & Drijvers, 2014).

This realistic situation underlies the implementation of PMRI since the beginning of learning.

Treffers (Heuvel-Panhuizen & Drijvers, 2014) formulated the main teaching principles of RME: 1) The principle of activities, which in RME students are positioned as active participants in the learning process. 2) The principle of reality which includes two ways i.e. a) it states the inherent importance of the objectives of mathematics education including the ability of students to apply mathematics in solving "real life" problems; b) it means that mathematics education should start from a problem situation that is meaningful to the student, which gives students the opportunity to attach meaning to the mathematical construction they develop while solving the problem. Instead of starting by teaching abstractions or definitions that will be applied later, in RME learning, learning is emphasized to start from problems with rich contexts that demand mathematical organization or can be mathematicalized process by students, placing students on the path of informal context solution strategies as the first step in the learning process.; 3) The principle of level, this means that in learning mathematics students graduate at different levels of understanding: from solutions related to informal contexts, through realizing different levels of shortcuts and schematicization, to gaining insight into how concepts and strategies are concerned. Models have an important role to play in bridging the gap between informal mathematics and more formal mathematics; 4) The intertwinement, this means that domains of mathematical content such as numbers, geometry, measurement, data handling are not considered isolated but highly integrated chapters of the curriculum. Students are offered a rich problem where they can use a variety of mathematical tools and knowledge; 5) The principle of RME interactivity indicates that learning mathematics is both an individual activity as well as a social activity. RME promotes discussion and group work that gives students the opportunity to share strategies and discoveries; 6) The guiding principle refers to Freudenthal's idea of the "guided reinvention" of mathematics, teachers must be proactive, educational programs should load design as leverage against shifts in student understanding.

The existence of the COVID-19 pandemic so that learning is carried out online requires students to become an autonomous, independent, character, and technology-literate learners. In a sense, with the majority of asynchronous learning, it is expected that students' mastery of mathematics is good. This breakthrough is an alternative by utilizing

the context of farm's equipment, which is thrasher and rice mill machines, with the consideration that the majority of the people of East Java Regency make a living as farmers can be used as a starting point to facilitate the construction of students' knowledge regarding composite function concepts, students are able to solve problems of daily life related to the material and become a person of character and culture.

METHOD

The study used design research as the method. Design research was a method for developing learning outcomes in the classroom through a reciprocal analysis of everything that happens in the classroom and its implementation (K. Gravemeijer & Cobb, 2006; Koeno Gravemeijer, 1994). The subjects in this study were 10 high school students. Subjects are randomly selected.

The steps of this research related to design research consisted of three-stage. Firstly, preliminary design, a study of the appropriate literature was carried out including the concept of composition function, learning syntax in PMRI, learning evaluation instruments, so that in this way a student hypothetical learning trajectory. Secondly, a design experiment was intended to collect data that was used to answer the research problem formulation. The design of learning activities was tested and improved at this stage. The learning design trials were given to several students of class XI science who were the research subjects. The results of this trial, it was used as a way to explore and make hypotheses related to students' mathematical thinking strategies in the material. Thirdly, retrospective analysis. At this stage, the data collected at the experimental design stage would be analyzed. The results of the analysis would be used for further design development. The hypothetical learning trajectory that had been arranged is used as a guide to answering research questions in the form of a description of the learning trajectory of students in learning the composition function by utilizing farm's tools such as power thrasher and rice mill machine.

RESULTS AND DISCUSSION

After researching by taking 10 students for research subjects and testing the hypothetical learning trajectory that has been compiled on the composition function material by utilizing the tool life which is thrasher machine and rice mill machine, the following results are obtained based on each stage of the research carried out.

Preliminary design

At the preliminary design, researchers have studied the literature review of the concept of composition function, learning syntax in PMRI, and learning evaluation instruments. Researchers applied initial concepts related to the use of farm tools such as thrasher and rice mill machines in learning material composite functions at the high school level. Hypothetical Learning Trajectory was developed in each composite function learning activity. This student learning trajectory was a mind map that would be passed by students towards understanding the concept of composition function. The learning trajectory used in this research consisted of understanding the concept of composite function with the context of farm's equipment "thrasher and rice mill machine", understanding the relationship that occurred between the two tools to determine the conditions for two functions to be composed. Students looked for the composition formula function and the conditions that must be met so that two functions could be composed, and the application of the composition function formula in everyday life problems. Figure 1 explained a learning trajectory for composition function learning at the high school level.

Based on the hypothetical learning trajectory in Figure 1. This resulted in the design of the student learning trajectory in learning the composition function as presented in Table 1.

Understanding the concept of composition function by using a thrasher and rice mill machine → The relationship between the input and output of the thrasher and rice mill machine to find the condition that two functions can be composed → Find the function composition formula based on the thrasher and rice mill machine working system → Find the function composition formula based on the thrasher and rice mill machine working system
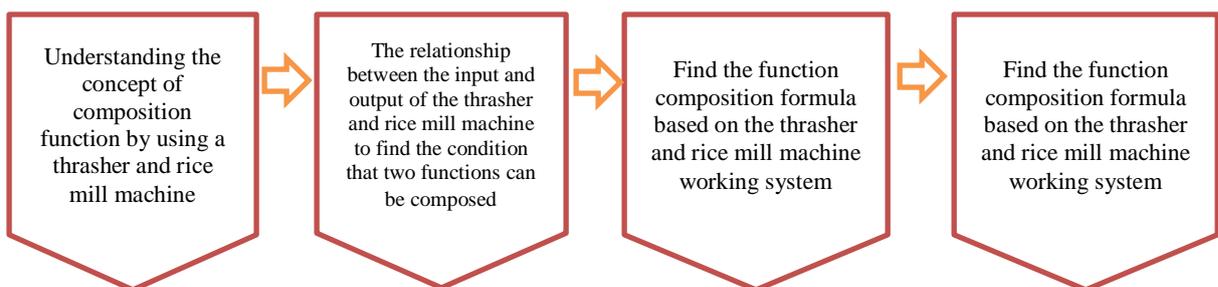
Figure 1. Learning Trajectory of Composition Function Learning in Senior High School

Table 1. Learning the Composition Function by Utilizing the Farm's Tool
"Thrasher and Rice Mill Machine"

Step	Learning Trajectory	Detail Activity
Context (informal)	Thrasher and rice mill machine	1. Student unknown the farm's tool; thrasher and rice mill machine 2. Students recognize and are familiar with the farm's tool 3. Students don't understand that in the thrasher and rice mill machine process there is a concept of function composition 4. Students realize that the use of thrasher and rice mill machines is related to the material composition function
Mode Of	Tool's working system	1. Students can identify the input, output, and working system of the thrasher and rice mill machine 2. Students have not been able to identify the input, output, and working system of the tool
Mode For	Terms of function composition (domain and range) through farm's tools	1. Students are able to determine the input and output of each tool and relate it to the rules that two functions can be composed 2. Students have not been able to determine the input and output of each tool and relate it to the rules that two functions can be composed 3. Students are able to make a representation of the relationship between the two tools associated with the concept of a function and its accompanying rules 4. Students haven't been able to make a representation of the relationship between the two tools associated with the concept of a function and its accompanying rules
Formal	Finding function composition formulas and implementing them in contextual problem solving	1. Students are able to find the formula for the composition of two functions 2. Students have not been able to determine the composition formula of two functions 3. Students are able to solve contextual problems related to function composition 4. Students have not been able to solve contextual problems related to function composition

Design Experiment and Analysis Retrospective

In this step, the researcher tested the learning trajectory that had been prepared for 10 eleventh-grade students from MAN 1 Ngawi (Senior High School). After that, the researcher made a retrospective analysis of the experimental activities (experimental

design) which were categorized into four activities, namely context, mode of, mode for, and formal.

The informal stage (context) carry out at this stage consist of 5 stages. It starts from (1) the teacher asking students to freely name farm's tools that they know, 2) the teacher asking students to describe farm's tools and their working systems and benefits, 3) the teacher asking students whether they have ever been involved in the process of harvesting rice and then understanding the process from rice to rice, 4) students observing pictures of farm's tools in the form of thresher and rice mill machines, and finally, 5) the teacher telling the history of technological developments in the form of farm's equipment to harvest rice starting from *ani-ani*, pounded with a pestle in a mortar, *herek* with manual pedaling, *herek* with machines, thrasher, bulldozer, then the process of grinding rice in rice mill machine in one village there may only be one rice mill post, developing into a lot of "Selep" motorbikes looking for customers. Selep is a local language for rise mill. The teacher's goal is to tell this revolution with the aim that students know the culture around them, appreciate local wisdom, and technological advances, so that cultured student characters are built. In the last activity, the teacher asks students whether the work system of the two machines is related to certain mathematical concepts.

In the step of Mode Of, after the teacher finished telling the story, then asked the students to observe and analyze the function and work system of each of the agricultural tools. Students were invited to express their opinions. Students were asked to mention how the process of separating rice stalks from rice. What was put in the thrasher machine? What was comes out? Then asked the students again about, how is the working system of the "selep" machine working? What material goes into the "selep"? Then what is comes out of the machine?

To strengthen the informal stage (context) and mode of, the teacher provides the following explanation to strengthen and clarify students' understanding in Figure 2.

In the Made for stage, students are directed to link and find the relationship between the input and output of the working system of these machines into a mathematical formula. To further relate it to the rule, two functions can be composed. Students are invited to make a representation of the relationship between the two tools associated with the concept of a composite function and its accompanying rules.

After the students are familiar with the two machines, then the teacher directs them to relate them to mathematical material, composite function. The teacher instructs how each machine works then pays attention to the input and output of each agricultural machine. Figure 3 is a student worksheet that facilitates the activity.

In the Formal Stage, the student activities at this stage are students who have understood the conditions for two functions to be composed and understand the function composition formula, students are given the teacher problems related to the material. The analysis carried out on the results of student work is mainly focused on what students do in solving the Composition of Function problem. Figure 4 presents question number one along with a sample of student answers.

Take a look at the illustration below!


Thrasher Machine   Rice Mill Machine

In a rice processing activity, to process rice plants into rice, Farmer has to use two machines. The first machine is thrasher, this machine functions to process and separate rice seeds from straw. The results of the first machine are rice seeds, *gabugan* , *awul-awul*, and *damen*. To get rice, only rice is taken, so a second machine is used, namely the selep machine. This selep machine is used to peel rice into rice. The results of this selep machine are rice, *bekatul*, and *brambut*. However, only rice is consumed.
a.   Make an illustration that explains the activity of the relationship between the two machines in the above situation!
b.   Give attention to the sequence of process steps in the problem above, then give your estimate of the definition of the composition function using the existing information!

Figure 2. Student Worksheet Related to the Initial Activity of Knowing Farm's Machinery and its Relation to Mathematics Concept

Well, the illustration above illustrates the composition of functions.
Now have you started thinking about what is function composition?
To understand composition operations on functions, consider the following figure below!
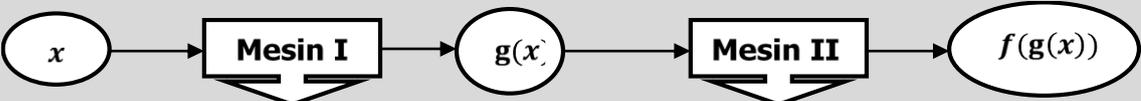
Pay attention to the sequence of steps in the process of processing raw materials into finished materials above. Relate the above information to the composition of the function then give an interpretation!
Let's together determine the function composition formula.
Given three of function they are f, g, and h.
1) Function $g : \mathbf{A} \to \mathbf{B}$
   Every element in $x \in D_g$ mapped to $y \in W_g$ with rule $y = g(x)$.
2) Function $f : \mathbf{B} \to \mathbf{C}$
   Every element in $y \in D_f$ mapped to $z \in W_f$ with rule $z = f(y)$.
3) Function $h : \mathbf{A} \to \mathbf{C}$
   Every element in $x \in D_h$ mapped to $z \in W_h$ with rule $z = h(x)$.
Give attention that function h is a mapping of the set A to set C with the intermediary set B. Function h is results form composition function g and function f i.e. composition function $(f \circ g)$, that $h = (f \circ g)$ or $h(x) = (f \circ g)(x)$.
Now the problem is how to state the formula for the composition function $(f \circ g)(x)$ in function $f(x)$ and function $g(x)$?
Function g determined by the rules : $y = g(x)$ .... (i)
Function f determined by the rules : $z = f(y)$ ....(ii)
Function h determined by the rules: $z = h(x)$ .... (iii)
Equation (ii) is same wih equation (i), (ii) = (i) .... (iv)
Subtitution for $y = g(x)$ from (i) to (iv), then :

Figure 3. Student Worksheet to Guide Students from Informal Math Concepts to Formal Math about Composite Function Concept

Know that two function, f and g where each function is determined by the formula:
$f(x) = 2x - 7, D_f = (-\infty,\ 15]$ and $g(x) = 10 - x, D_g = [-6, \infty)$.
a. Investigate what is the function of composition $(f \circ g)(x)$ dan $(g \circ f)(x)$ can be formed to composition function? Explain your reasons!
b. If there is one of $(f \circ g)(x)$ or $(g \circ f)(x)$ which can not be formed as a function, how do you make the two functions can be composed?

Figure 4. The Questions to Explore Students' Understanding about the Material Requirements for Two Functions to be Composed

The question above aims to explore students' understanding related to the condition that two functions can be composed. Students investigate each function by checking the domain and range of each given function. Identify which of the $(f \circ g)(x)$ and $(g \circ f)(x)$ can be composed then if one cannot be composed, students are asked to write down the reason then look for ways how to make the two functions can be composed.

To investigate whether these two functions can be composed, students must understand the terms that the two functions can be composed, namely $(f \circ g)(x)$ if $R_g \cap D_f \neq \emptyset$ and $(g \circ f)(x)$ jika $R_f \cap D_g \neq \emptyset$, The following is a sample of student answers related to the question above in Figure 4.

Figure 5 gives the information that students are able to determine the initial steps to solve a given problem, namely by writing down the rules and conditions for two functions to be composed. However, in the sample, students did not seem to understand the material, this was reflected in the students who included the requirements $(f \circ g)(x)$ is formed if $R_g \cap D_g \neq \emptyset$, $D_f = (-\infty, 15)$ and $g(x) = 10 - x$, $D_g = (-6, \infty)$. Students tend not to care about the domain area of the function. Errors still continue, namely in determining R_f, students do not pay attention to the domain of the function f. Then the students did not conclude whether $(f \circ g)(x)$ and $(g \circ f)(x)$ can be formed or not. In addition, there is no mathematical communication in the solution, students seem very unfocused on the existing problems. Thus the students in the sample have not been able to construct the information in the problem. Students only react with personal will and do not try to compare and contrast with a general principle. So that the problem-solving process that should be passed by students, just passes.

Figure 5 is only one sample of student answers that were used as research subjects in this study. Students fail to answer questions because students do not focus on the problem, students react directly and do not confirm whether the written solution is in accordance with the information and composite function rules. From this situation, it is the teacher's job to provide scaffolding to students according to the stage of students' cognitive development. The teacher can ask students to re-examine the problem, re-express the rules or conditions for the composition of functions and check the computations.

Figure 5. Student answer for Question Student Worksheet

Based on all stages of research design that has been done it can be known that students are able to make associations from thrasher machines and rice milling through the determination of machine inputs and outputs to the formulation of composite function concepts. The use of the context of thrasher machine and rice milling is in line with the use of Kawung Batik pattern for learning transformation geometry (rotation) (Risdiyanti & Prahmana, 2018). Through batik pattern Kawung students can understand the concept of rotation. It is also in line with batik pattern for learning reflection material (Novrika et al., 2016), use of card games in learning number operations (Prahmana, 2012), and rotating card game for learning Lowest Common Multiple (LCM) concept (Prahmana, 2010).

Stages in the learning trajectory have an important role in understanding the concept of compositional functions from the informal level to the formal level. The "learning trajectory" helps diagnose a student's level of mathematical understanding and provides guidance for teaching (D. Clements & Sarama, 2014). Geometric curricula for young people are most effective when covering a wide range of tasks based on a diverse and non-sample learning trajectory, nurturing visual cognition with development towards analytical thinking, integrating rich and diverse mathematical speech (D. H. Clements et al., 2018). This is in accordance with the use of Math GASING which has a real contribution to students in understanding and mastering the concept of multiplication operations (Hendriana et al., 2019). The study also explains the strategies and models found by students in learning multiplication that students use as the basic concept of multiplication; the students can understand the concept of multiplication more easily, and they show interest in using this learning trajectory (Hendriana et al., 2019). Learning trajectory construction is used in the manufacture of the math-mapper 6-8 tool, to help the scaffold curriculum towards improving student-centered coherence(Confrey et al., 2017).

CONCLUSION

The results of this study indicate that learning the composite function can be integrated by using agricultural equipment such as a thrasher and rice mill. Through student worksheets, students are directed to observe the working system of the two tools, the input process, and the output. After that, students are directed to link informal mathematical concepts from the work system of these two tools into formal mathematical concepts. How two functions can be composed, the conditions for two functions to be composed, considering the domain (D_f) and range (R_f) of each function, and what if the two functions can not be composed are explained in this study. The involvement of everyday contexts in learning mathematics helps students understand and construct students' mathematical knowledge because context facilitates students in the process of mathematical imagination and abstraction. This research is in line with previous studies that have been carried out that support the presence of mathematics education reform through the Indonesian Realistic Mathematics Education, which emphasizes two aspects of transmitting mathematical material as mathematical activity and the opportunity for students to reinvent (reinvention) through practice.

This conclusion can be used to give some advice. The first advice for math teachers is that teachers should be able to take advantage of the use of thrasher machines and rice milling in order to understand students about determining inputs, outputs and composite function concepts. This can be implemented at the learning planning stage by compiling appropriate learning tools, at the learning implementation stage, and at the assessment stage of learning outcomes. The second suggestion is for researchers in the field of mathematics education is to carry out research design using everyday context in different learning materials or using different contexts on the same material. It is intended that the world of mathematics education has a rich learning alternative by utilizing everyday context.

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